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CONSERVATION NUMBERS IN THE PETRI NET MODEL OF THREE-PHASE TRAFFIC LIGHTS WITH THE NORWEGIAN SYSTEM

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ABSTRACT. *The Petri net model can be used to represent the discrete-state behavior of traffic light signals. The Petri net model can also model three-phase traffic light synchronization. This study aims to examine the conservation number of the three-phase traffic light Petri net model that implements the Norwegian system. The Petri net model must pass validation and verification tests, including boundedness, conservation properties, coverability for all states, and simulation. The Petri net model also includes Place Invariants, which represent the dynamic system behavior that remains constant over time. The study results indicate that the model complies with all Place-Invariants, meets all required properties, and the simulation is also accurate. Place-Invariants in each phase must include a dummy. The three-phase traffic-light Petri net model with the Norwegian system has a conservation number that holds for the model as a whole. The conservation number is an extension of Place-Invariant that only applies partially to the model.*
Keywords*: Petri net model, traffic light signals, Norwegian system, place-invariants, conservation number.*

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ABSTRAK. Model Petri net dapat digunakan untuk merepresentasikan struktur perilaku *state diskrit* dari sinyal lampu lalu lintas. Model Petri net juga mampu menyajikan singkronisasi penjadwalan tiga fase lampu lalu lintas. Studi ini bertujuan untuk mengkaji bilangan konservasi pada model Petri net lampu lalu lintas tiga fase yang mengimplementasikan sistem Norwegia. Model Petri net harus memenuhi uji validasi dan verifikasi, diantaranya menggunakan properti keterbatasannya (*boundedness*), konservasi (*conservation*), *coverability* untuk semua keadaan/ *state*, dan simulasi. Model Petri net juga memiliki *Place-Invariant* sebagai representasi dinamika perilaku yang senantiasa terjaga konstan sepanjang waktu. Hasil studi menyatakan bahwa model sesuai dengan semua Place-Invariant, memenuhi semua properti yang disyaratkan, dan simulasinya juga tepat. Place-Invariant pada masing – masing fase harus menyertakan dummy. Model Petri net lampu lalu lintas tiga fase dengan sistem Norwegia memiliki sebuah bilangan konservasi yang berlaku untuk model secara keseluruhan. Bilangan konservasi merupakan pengembangan Place-Invariant yang hanya berlaku secara parsial pada model.

Kata Kunci: model Petri net, sinyal lampu lalu lintas, sistem Norwegia, *place-invariant*, bilangan konservasi.

1. INTRODUCTION

Petri nets are a tool for graphically modeling a system's behavioral structure. Petri nets can be used to model and analyze the performance of manufacturing systems (Taiping & Peisi, 2015). It is used in a multi-channel queuing system scheduling (Pramesthi, 2018). Petri nets can also be used to model traffic lights (Adzkiya, 2008),(Cahyono et al., 2019),(Tristono et al., 2018).

Traffic lights regulate traffic at road junctions or intersections with multiple arms. The scheduling interval time must be appropriate to the volume of vehicles passing through each arm of the road. Traffic lights must ensure that the flow from each road arm is safe and does not conflict with the movement of vehicles from other directions. Scheduling using traffic lights must also not cause new congestion (Tamin, 2000).

The Norwegian traffic light system is slightly different from the standard system. The standard system's signal order is green, yellow, red, and then returning to green as the initial state. The yellow and red signals in the Norwegian system are lit simultaneously when the signal is ready to return to green again. The two signals serve to reduce travel delays and also to prevent conflicts. When the signals are red and yellow, vehicles that have been stopped for a long time can prepare to move forward again (Tristono et al., 2023).

The Norwegian system can also be modified. The Norwegian system's red-yellow signals are converted to a single yellow signal. In each traffic light cycle of the modified Norwegian system, the yellow signal flashes twice: before and after the red signal (Tristono et al., 2024). Regarding the problem of traffic light signal control structure complexity, the Norwegian system is the highest among the Standard and Modified Norwegian systems (Tristono et al., 2019).

This study aims to investigate the conservation number of the three-phase traffic-light Petri net model implementing the Norwegian system. Based on the definition, the conservation number of a Petri net can be used to test the validity of the condition for every possible state (Cassandras, 1993). It is difficult when the number of states to be tested is large, and impossible when the number is infinite (Adzkiya, 2008). As methods, first, the Petri net model validation and verification is tested using the properties. The coverable tree is constructed based on the occurrence matrix formula and the fire transition. Next, using the constructed invariant, the conservation number is determined. This study has never been done before.

In this study, the phases of traffic lights have a fixed order. Over time, the phase sequence is west, north, east, and back to west as the initial. Traffic light phases are a sequence of signals that alternately illuminate to regulate traffic flow from different arms of an intersection. Each phase prevents conflicts between vehicles coming from different directions (Tamin., 2000).

The studied model employs a fixed time strategy. It is a strategy for setting the time interval of traffic light signals based on past empirical data from the volume of passing vehicles. Each signal lights up at a specific time interval that remains constant throughout (Tristono et al., 2023). Traffic lights have at least three discrete state spaces. These state spaces are represented by colors: red, yellow, or green. These three signals light up at fixed time intervals and in a specific order, alternating and repeating, thereby creating a repeating cycle of the traffic light state space (Tristono et al., 2019),(Tamin., 2000).

Many previous studies have analyzed traffic light behavior using Petri nets. The study developed a timed Petri net (TPN)-based controller to mitigate traffic congestion that leads to accidents. It uses adaptive traffic light sequences in real time (Row et al., 2024). Petri Net (PN)-based approaches are used to model deadlock formation analytically and have been applied in many traffic flow studies. PN is used to investigate the probability and duration of deadlock formation (Qi et al., 2022). A new hybrid Petri net model for traffic behavior at intersections is used to investigate flow dynamics in urban networks (Vázquez et al., 2010). Research on control and optimization methods for intersection traffic signals based on timed colored Petri net (TCPN) (Li et al., 2019). A new extension of hybrid Petri nets is presented to generalize traffic flow modeling by accounting for state dependence on both time-deterministic and nondeterministic external rules, such as stop signs or priority roads. Hybrid Petri nets, which consider both discrete and continuous aspects of traffic flow dynamics, have proven to be a powerful tool for approximating these dynamics and accurately describing vehicle behavior (Riouali et al., 2016).

2. RESEARCH METHODS

The Petri net model validation and verification tests employed the boundedness property, conservation, coverability for all states, and simulation. The boundedness property is a guarantee that the number of tokens in a place does not explode to infinity. The number of tokens in the model's places cannot exceed a certain positive integer. The conservation property states that the marking of the Petri net model is sustainable, that there are no dead-end states due to the absence of enabling transitions, and that it can be repeated to return to the initial state. The meaning of the coverability property is that all enabled transitions must be part of the sequence of state trees; no transitions or states are missed, meaning that all transitions and states are covered (Adzkiya, 2008). The simulation illustrates the real-life event of a traffic light system signal sequence turning on.

The Petri net model validation and verification tests also use several Place Invariants (Tristono et al., 2021). The invariants are used to determine structural properties of the net regardless of a specific Petri net marking. The invariants are reduced to the solution of systems of linear homogeneous equations in integer non-negative numbers (Zaitsev, 2004). A Linear Time-Invariant (LTI) system is a dynamic system whose components remain constant over time (Safitri, 2019). Place-Invariant is a tool for verifying and validating the correctness of a Petri net model. Place-Invariant presents the similarity of markings across multiple places in a Petri net model at a specific integer (Tristono et al., 2021).

After the Petri net model validation and verification tests using the property, coverable tree is constructed based on the occurrence matrix formula and the fire transition then the invariant is constructed, and the conservation number is found.

2.1 Petri Net

This study uses four components of a basic Petri net for modeling traffic lights: places (P), represented as circles; transitions (T), represented as rectangles; directed arcs; and tokens. A place (P) represents the state space that occurred. A transition (T) can be enabled or not enabled. The enabled transition is used to trigger an initial state and transform it into the next state according to the arc direction. A state that occurred is indicated by the presence of a token in the associated place (Adzkiya, 2008).

The basic Petri net model is N(P, T, A, w). The finite set of places are P = {P1, P2, . . . , Pm} and the finite set of transitions are T = {t1, t2, . . . , tn}, where m, n are positive integers. Component A is the set of arcs that relate places to transitions or vice versa, i.e., from transitions to places. The mathematical representation is written as $A \subseteq (P \times T) \cup (T \times P)$. Component w (weight) represents the weight function of an arc. The value of w is $A \to \{1, 2, 3, . . . \}$ (Adzkiya, 2008). Arc weights in the model, which have a value of one, are usually omitted and represented by directed lines.

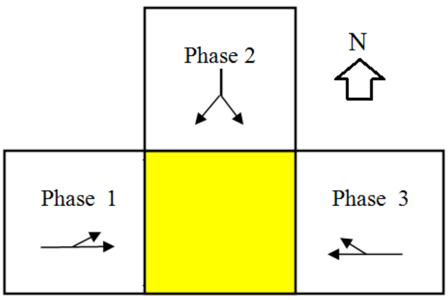

Figure 1. Three Phases of Traffic Lights and Their Flow

Marking a Petri net place uses tokens, which can be 0 or 1. They respectively mean off or on. Mathematically, it is written M(Pi) = {0, 1}, where i = 1, 2, 3,..., 12. In the three-phase Norwegian traffic light model discussed, there are only 12 places. A place in the traffic light signal model is said to be marked or on if a token is present there. A place for a signal will be empty if the signal in question is not on. To draw the three-phase traffic light model of the Norwegian system, Petri Net Simulator 2.0 was used.

2.2 Petri Net Model of Norwegian Traffic Light System with Three Phases

A junction consisting of three road segments: the west, north, and east. The traffic light settings are as follows: Phase 1 is the schedule for traffic from the west, Phase 2 is the schedule for traffic flow from the north, and Phase 3 is the schedule for traffic flow from the east. The vehicle flow system is left-hand traffic. There are no pedestrians at the road junction, so there is no need to allocate time for it.

Figure 2 illustrates a Petri net model of the Norwegian traffic light system, which consists of three phases. In the west arm of the traffic light model, the signals are P1, P2, and P3, which are green, yellow, and red, respectively. In the north arm of the traffic light model, there are signals P5, P6, and P7. Furthermore, in the east arm of the traffic light model, there are signals P9, P10, and P11. Places P4, P8, and P12 are intermediate places to build the signal sequence, according to the Norwegian system.

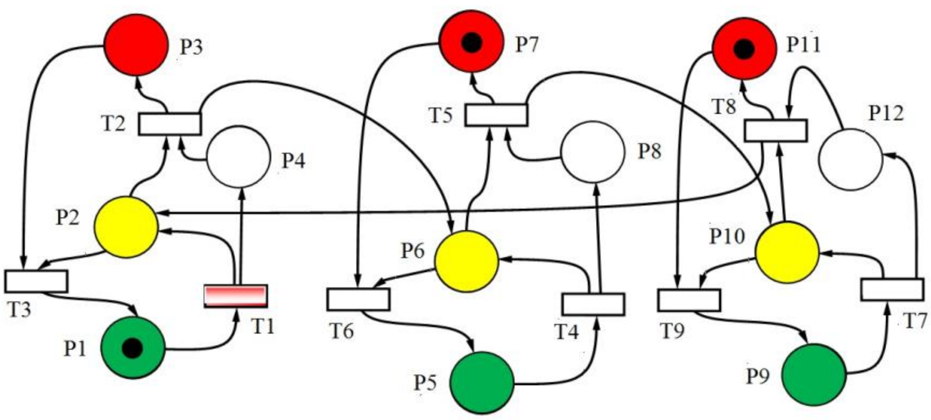

Figure 2. Petri Net Model of a Three-Phase Traffic Light Using the Norwegian System with T1 Enabled Transition.

After a token is fired consecutively at any of the P4, P8, or P12 by transitions T2, T5, or T8, the red signal for all three traffic lights turns red, commonly known as an all-red signal. At the end of each traffic light phase change, an all-red signal is always present. The purpose is to provide a brief period for vehicles to clear the intersection (Tamin., 2000).

The T2, T5, or T8 transitions will synchronize the three traffic light phases. It will cause the yellow signal to illuminate simultaneously with the red signal, in phases 2, 3, and 1, respectively. Vehicles on the junction arm whose yellow signal illuminates simultaneously with the red signal can prepare to resume their travel after a long stop. This signal is designed to ensure safe travel and prevent traffic conflicts (Tamin., 2000).

If there is a token at place P1, as shown in Figure 2, then the green signal for the traffic light on the west side of the road is on. Consequently, the signals on the other two sides of the road must be red. While places P7 and P11 each have one token, or both are lit.

The results of the traffic light schedule simulation in Table 1 are illustrated in Figure 3. Each green signal duration is 30 seconds in phase 1, 21 seconds in phase 2, and 30 seconds in phase 3. The duration of one traffic light cycle is 99 seconds. This duration is determined based on empirical data on the volume of passing vehicles and remains constant over time.

Table 1. Traffic Light Schedule

Phase	Green	InterGreen		Red	Cycle
		Yellow	AllRed		
Second					
1.	30	3	3	66	99
2.	21	3	3	75	99
3.	30	3	3	66	99

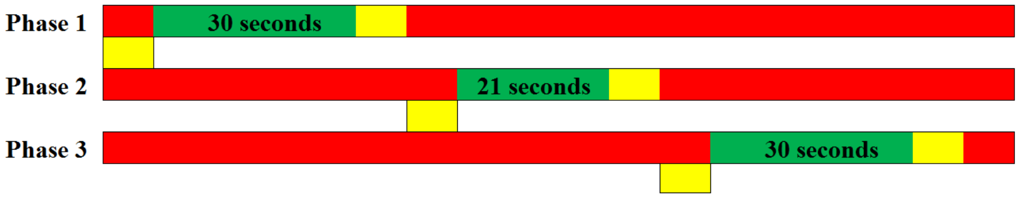

Figure 3. Simulation results of the Petri net model of Three-Phase Traffic Lights with the Norwegian System.

2.3 Incidence Matrix

A forward incidence matrix and a backward incidence matrix represent the connectivity in a Petri net. The incidence matrix of the Petri net model is shown in Figure 4. The matrix A is the difference between the backward incidence matrix and the forward incidence matrix. The matrix is m x n, where m represents the number of places and n the number of transitions. The values of m and n are non-negative integers. The forward incidence matrix elements are the weights of the arcs connecting transitions to places. It also means that places are the outputs of transitions. The backward incidence matrix elements are the weights of the arcs connecting places to transitions. It means that places are the inputs of transitions. If there are no arcs connecting places to transitions or vice versa, then the arc weight is assigned a value of zero (Adzkiya, 2008).

3. RESULTS AND DISCUSSION

Equation (1) is the transition matrix from T1 to T9. The matrices T1 to T9 are presented in their transposed form. The incidence matrix, A, is shown in Figure 4. Equations (2) through (5) are formulas in the form of an incidence matrix after firing by an enabling transition. Its implementation is by multiplying the matrices. A transition is enabled if all its input places contain tokens greater than the arc weight to the next destination place (Adzkiya, 2008). A transition fires once per traffic light cycle and must not miss any :

$$\left.\begin{aligned}
T1 &= [\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ]^T\\
T2 &= [\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ]^T\\
T3 &= [\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ ]^T\\
T4 &= [\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ ]^T\\
T5 &= [\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ ]^T\\
T6 &= [\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ ]^T\\
T7 &= [\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ ]^T\\
T8 &= [\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ ]^T\\
T9 &= [\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ ]^T
\end{aligned}\right\} \qquad (1)$$

Here is the formula for the occurrence matrix :

$$O_{i+1} = O_i + A.T_i\ , \quad i = 1, 2, 4, 5, 7, 8 \tag{2}$$

$$O_4 = O_3 + A.T_6 \tag{3}$$

$$O_7 = O_6 + A.T_9 \tag{4}$$

$$O_1 = O_9 + A.T_3 \tag{5}$$

A =

	T1	T2	T3	T4	T5	T6	T7	T8	T9
P1	-1	0	1	0	0	0	0	0	0
P2	1	-1	-1	0	0	0	0	1	0
P3	0	1	-1	0	0	0	0	0	0
P4	1	-1	0	0	0	0	0	0	0
P5	0	0	0	-1	0	1	0	0	0
P6	0	1	0	1	-1	-1	0	0	0
P7	0	0	0	0	1	-1	0	0	0
P8	0	0	0	1	-1	0	0	0	0
P9	0	0	0	0	0	0	-1	0	1
P10	0	0	0	0	1	0	1	-1	-1
P11	0	0	0	0	0	0	0	1	-1
P12	0	0	0	0	0	0	1	-1	0

Figure 4. Incidence Matrix of the Three-phase Light Petri Net Model with the Norwegian System

Figure 5 illustrates the sequence of occurrences in the three-phase traffic light Petri net model that implements the Norwegian system. Nine occurrences/ events are combined in a row into a 12 x 9 occurrence matrix. It is called the Occurrence matrix or O.

O =

	O1	O2	O3	O4	O5	O6	O7	O8	O9
P1	1	0	0	0	0	0	0	0	0
P2	0	1	0	0	0	0	0	0	1
P3	0	0	1	1	1	1	1	1	1
P4	0	1	0	0	0	0	0	0	0
P5	0	0	0	1	0	0	0	0	0
P6	0	0	1	0	1	0	0	0	0
P7	1	1	1	0	0	1	1	1	1
P8	0	0	0	0	1	0	0	0	0
P9	0	0	0	0	0	0	1	0	0
P10	0	0	0	0	0	1	0	1	0
P11	1	1	1	1	1	1	0	0	1
P12	0	0	0	0	0	0	0	1	0

Figure 5. Occurrence Matrix of the Petri Net Traffic Light Model

The sequence of fire transitions and their occurrences is shown in Figure 6. The occurrences/ events start from [O1] → T1, namely with the enable transition T1, and repeat back to [O1] → T1. All transitions in the model have fired at least once during one traffic light cycle.

The sequence of transitions that fire and repeat themselves forms a traffic light cycle, and there is never a deadlock. It proves the conservation property. The Petri net for the traffic light model meets the liveness property. The number of tokens in place does not expand and continues to grow to infinity. It means the model also satisfies the boundedness property. Coverability for all states is also satisfied because it forms a repeating cycle.

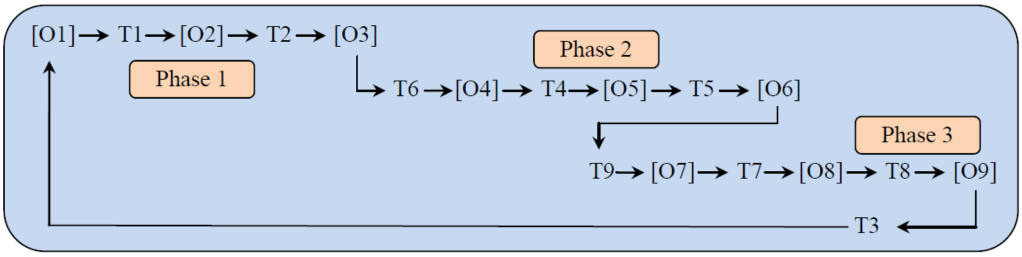

Figure 6. Fire sequence of transitions and their occurences to prove the coverability tree

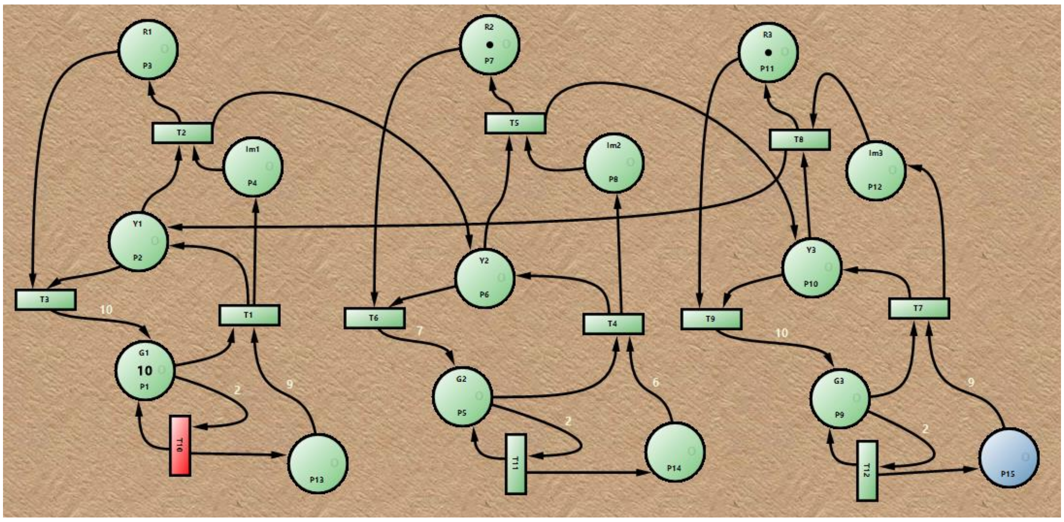

Figure 7. Petri Net Model of a Traffic Light with Three Phases equipped with time

The model with the studied fixed-time strategy can be extended to simulate traffic lights with complex timing at each signal. An illustration is shown in Figure 7. Attributes G1/ P1, Y1/ P2, and R1/ P3, respectively, represent the

Green, Yellow, and Red signals in phase 1 of the traffic light. Similarly, attributes G2/ P5, Y2/ P6, and R2/ P7 represent phase 2, and G3/ P9, Y3/ P10, and R3/ P11 represent phase 3.

The arc weight from transition T3 to P1 is 10. It is assumed that each token has a duration of three seconds. It means that the green signal in phase 1 has an interval of 30 seconds, as shown in Table 1. Similarly, the arc weight from transition T6 to P4 is 7. The green signal in phase 2 has an interval of 21 seconds. Furthermore, the green signal in phase 3 has an interval of 30 seconds because the arc weight from transition T9 to P7 is 10.

The all-yellow and all-red signals last three seconds, so they are illustrated with a single token. Durations do not accompany all red signals. It is because the Petri net model is synchronized with the green signal on all road arms from the junction. It ensures that no errors will occur in the traffic light model. All red signals do not need to be accompanied by durations. Their time intervals are automatically structured. It is one of the advantages of the Petri net model.

Invariant, this time, means Place-Invariant, namely a Linear Time Invariant (LTI) system in a dynamic place marked by tokens that do not change over time (Safitri, 2019). Invariant and simulation results are included as properties that can be used for validation and verification to ensure a model is correct. Traffic lights must be able to regulate **traffic flow** at junctions, avoid conflicts in vehicle movement, cover all signal phases on the road arm, and allow for a return to the initial position to create a traffic light cycle (Adzkiya, 2008).

Linear Time-Invariant (LTI) systems in the Norwegian traffic light system are described by Invariants (1) to (9). Invariant (1) that does not change over time applies to phase 1 traffic lights. A token may be at P1 or P2 when there is a token at P7 and P11. The traffic light signal in phase 1 (phase west) may turn green or yellow if the signals on the other two arms, namely the north arm and the east arm, are both always red or M(P7)= M(P11)= 1. Invariant (1) is a guarantee that the travel of vehicles coming from the west arm does not conflict with other traffic coming from different road arms. Invariant (2) applies to phase 2 traffic lights, and Invariant (3) applies to phase 3 traffic lights :

$M(P1)+M(P2)=M(P7)=M(P11)$ $\quad$ *when* $M(P7)=M(P11)=1$ $\quad$ Invariant (1)

$M(P5)+M(P6)=M(P3)=M(P11)$ $\quad$ *when* $M(P3)=M(P11)=1$ $\quad$ Invariant (2)

$M(P9)+M(P10)=M(P3)=M(P7)$ $\quad$ *when* $M(P3)=M(P7)=1$ $\quad$ Invariant (3)

Invariant (4) describes the synchronization for the three path arms at the junction to share the travel schedule time. M(P1) and M(P2) are the green and yellow signal places for phase 1. M(P5) and M(P6) are the green and yellow signal places for phase 2. M(P9) and M(P10) are the green and yellow signal places for phase 3. The three-phase traffic light Petri net model with the Norwegian system satisfies Invariants (1) through (4).

$M(P1)+M(P2)+M(P5)+M(6)+M(9)+M(P10)=1$ $\quad$ Invariant (4)

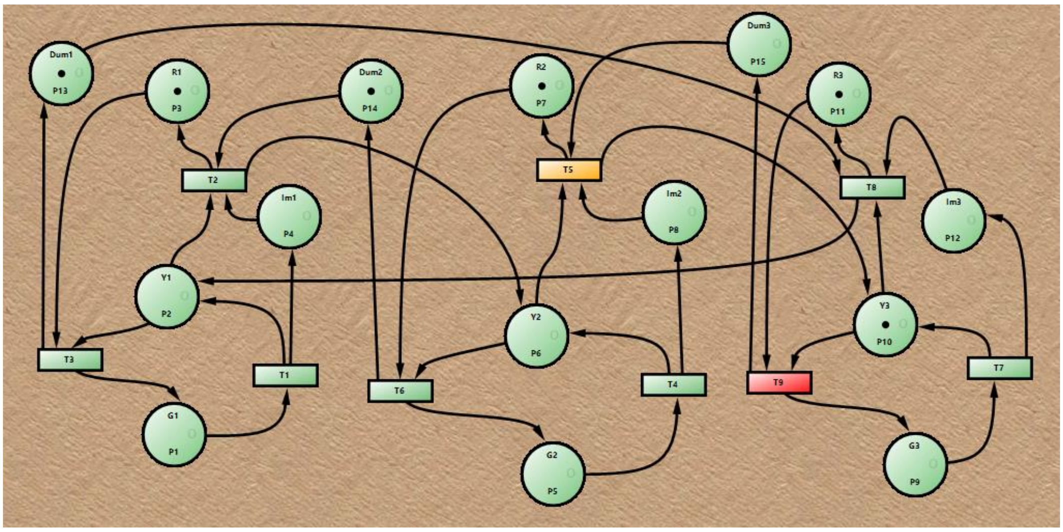

Figure 8. Petri Net Model of Traffic Lights with Three Phases, equipped with the Dummy Places, namely P13, P14, and P15

Place P13 in Figure 8 is a dummy place that serves as the signal duration counter, except during simultaneous red and yellow signals in phase 1. Similarly, P14 and P15, respectively. Next, Invariant (5) to Invariant (7) can be constructed. Invariant (5) applies to phase 1, Invariant (6) is used in phase 2, and Invariant (7) is implemented in phase 3. All total Marking sums are worth 2. Invariants (5) to (7) cannot be constructed without the dummy places, namely P13, P14, and P15.

The number of places in the model using the dummies increases by three, while the number of transitions remains the same. Similarly, the firing sequence of the transitions in the three-phase Petri net traffic light model with the dummy remains unchanged from the model without the dummy. However, there are slight changes in the actual occurrences. The Incidence Matrix for the model using dummy is shown in Figure 9.

A1 =

	T1	T2	T3	T4	T5	T6	T7	T8	T9
P1	-1	0	1	0	0	0	0	0	0
P2	1	-1	-1	0	0	0	0	1	0
P3	0	1	-1	0	0	0	0	0	0
P4	1	-1	0	0	0	0	0	0	0
P5	0	0	0	-1	0	1	0	0	0
P6	0	1	0	1	-1	-1	0	0	0
P7	0	0	0	0	1	-1	0	0	0
P8	0	0	0	1	-1	0	0	0	0
P9	0	0	0	0	0	0	-1	0	1
P10	0	0	0	0	1	0	1	-1	-1
P11	0	0	0	0	0	0	0	1	-1
P12	0	0	0	0	0	0	1	-1	0
P13	0	0	1	0	0	0	0	-1	0
P14	0	-1	0	0	0	1	0	0	0
P15	0	0	0	0	-1	0	0	0	1

Figure 9. Petri Net Incidence Matrix of the Norwegian Traffic Light System Model with the Dummy Places

$$M(P1)+M(P2)+M(P3)+M(P13)=2 \qquad \text{Invariant (5)}$$

$$M(P5)+M(P6)+M(P7)+M(P14)=2 \qquad \text{Invariant (6)}$$

$$M(P9)+M(P10)+M(P11)+M(P15)=2 \qquad \text{Invariant (7)}$$

The simulation results are shown in Figure 11. The results differ slightly from the simulation results in Figure 3. Invariant (5), Invariant (6), and Invariant (7) guarantee that the traffic light signals in each phase are always active.

O1 =

	O11	O12	O13	O14	O15	O16	O17	O18	O19
P1	1	0	0	0	0	0	0	0	0
P2	0	1	0	0	0	0	0	0	1
P3	0	0	1	1	1	1	1	1	1
P4	0	1	0	0	0	0	0	0	0
P5	0	0	0	1	0	0	0	0	0
P6	0	0	1	0	1	0	0	0	0
P7	1	1	1	0	0	1	1	1	1
P8	0	0	0	0	1	0	0	0	0
P9	0	0	0	0	0	0	1	0	0
P10	0	0	0	0	0	1	0	1	0
P11	1	1	1	1	1	1	0	0	1
P12	0	0	0	0	0	0	0	1	0
P13	1	1	1	1	1	1	1	1	0
P14	1	1	0	1	1	1	1	1	1
P15	1	1	1	1	1	0	1	1	1

Figure 10. Petri Net Traffic Light Model Event with the Dummy Places

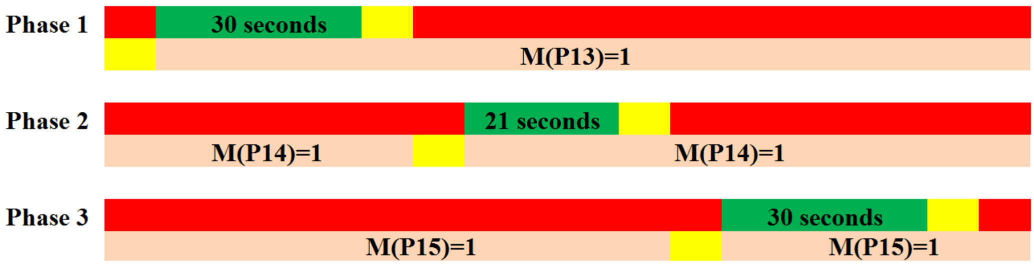

Figure 11. Simulation Results of the Model Equipped with the Dummy Places

Based on the verification, the Norwegian three-phase traffic light system model was declared valid despite the addition of the dummy places. The firing sequence of the transitions in the three-phase Petri net model of the traffic light with the dummy places remained unchanged. It means the model has a state space coverability tree that can revert to the initial state to form a traffic light cycle.

$$2M(P1)+M(P2)+M(P3)+M(P4)+2M(P5)+M(P6)+M(P7)+M(P8)+ 2M(P9)+M(P10)+M(P11)+M(P12)=4 \quad \text{Invariant (8)}$$

$$2M(P1)+M(P2)+2M(P3)+M(P4)+2M(P5)+M(P6)+2M(P7)+M(P8)+ 2M(P9)+M(P10)+2M(P11)+M(P12)+M(P13)+M(P14)+M(P15)=9 \quad \text{Invariant (9)}$$

Invariant (8) is a conservation number on the three-phase Petri net model of the traffic light in the original model. The conservation number value in the original model is 4. Invariant (9) is a conservation number on the three-phase Petri net model of the traffic light with the dummy places. The conservation number value in the model with the dummy places is 9. Invariants (8) and (9) also provide conservation numbers that guarantee that the three-phase Norwegian traffic light cycle can continue sustainably. A conservation number in the three-phase Petri net model of the modified Norwegian traffic light can be easily determined for each phase. It did not need the dummy places (Tristono et al., 2024).

4. CONCLUSION

The study results show that the model meets all required properties, the simulation is accurate, and obeys all Place-Invariants. Coverability tree for all states is also satisfied because it forms a repeating cycle. Place-Invariants in each phase must include the dummies. The three-phase Petri net model of the

Norwegian traffic light system has a conservation number that holds for the model as a whole. This conservation number extends Place-Invariant, which only partially applies to the model.

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