De Broglie Wave Analysis of the Heisenberg Uncertainty Minimum Limit under the Lorentz Transformation

Fima Ardianto Putra
Department of Physics, Universitas Jenderal Soedirman,
Jl. Dr. Soeparno 61, Kampus UNDOED Karangwangkal, Purwokerto, 53123 - INDONESIA
Email: ardiantoputraf2@gmail.com

Abstract - A simple analysis using differential calculus has been done to consider the minimum limit of the Heisenberg uncertainty principle in the relativistic domain. An analysis is made by expressing the form of Δx, Δt, Δp, and ΔE based on the Lorentz transformation, and their corresponding relation according to the de Broglie wave packet modification. The result shows that in the relativistic domain, the minimum limit of the Heisenberg uncertainty is $\Delta p\Delta x \geq \gamma^2\hbar/2$ and/or $\Delta E\Delta t \geq \gamma^2\hbar/2$, with γ is the Lorentz factor which depend on the average/group velocity of relativistic de Broglie wave packet. While, the minimum limit according to $\Delta p\Delta x \geq \hbar/2$ or $\Delta E\Delta t \geq \hbar/2$, is the special case, which is consistent with Galilean transformation. The existence of the correction factor signifies the difference in the minimum limit of the Heisenberg uncertainty between relativistic and non-relativistic quantum. It is also shown in this work that the Heisenberg uncertainty principle is not invariant under the Lorentz transformation. The form $\Delta p\Delta x \geq \gamma^2\hbar/2$ and/or $\Delta E\Delta t \geq \gamma^2\hbar/2$ are properly obeyed by the Klein-Gordon and the Dirac solution.

Keywords: De Broglie wave packet, Heisenberg uncertainty, Lorentz transformation, and minimum limit.

INTRODUCTION

The Heisenberg uncertainty principle, which is mathematically expressed as $\Delta p\Delta x \geq \hbar/2$ and/or $\Delta E\Delta t \geq \hbar/2$, is a fundamental principle in the quantum domain. This Principle restricts simultaneous measurements of *p*-*x* and *E*-*t* pair to be not less than *ħ/2* [1-4], which is the smallest limit of the Heisenberg uncertainty. The interesting point according to the authors' viewpoint, is that the forms $\Delta p\Delta x \geq \hbar/2$ and/or $\Delta E\Delta t \geq \hbar/2$ do not consider whether the observations are done in the non-relativistic or in the relativistic domain, inspite of the fact that each domain obeys different transformation rule. In the non-relativistic domain, quantities *p*, *x*, *E*, and *t* have to be consistent with Galilean transformation and in the relativistic domain, these quantities have to be consistent with the Lorentz transformation[2]. It is not evident if the form $\Delta p\Delta x \geq \hbar/2$ and/or $\Delta E\Delta t \geq \hbar/2$ are actually consistent with both Galilean and Lorentz transformation or with either one of these transformations.

There were speculations that when the Heisenberg uncertainty relation was built, there had been the Einstein-deBroglie relation,which had taken into account the relativistic effect [5-8]. But the relativistic effect did not seem to appear explicitly in the Heisenberg uncertainty relation, as far as we know. In Relativistic Quantum Theory, we can find uncertainty relations $m_0 c\lambda_c \geq \hbar$ and/or $m_0 c^2\tau \geq \hbar$ [9,10]. But it does not give explanations about time dilation and length contraction, while these phenomena do happen, for example in the case of muon particle when it is moving along the earth atmosphere [2].

In this paper, we propose a hypothesis that if quantities p, x, E, and t are subject to a different transformation rule, it should result in a different form of their corresponding uncertainties. This is the question we seek to answer, i.e. whether or not distinct form of the Heisenberg uncertainty exist for each non-relativistic and relativistic domain. To address that point, in this paper we will subject Δp, Δx, ΔE, and Δt to the Lorentz transformation. The condition for Δp, Δx, ΔE and Δt can be manifested in the picture of de Broglie wave packet explicitly [3]. Based on that viewpoint, whether or not the form of Heisenberg $\Delta p\Delta x \geq \hbar/2$ and/or $\Delta E\Delta t \geq \hbar/2$ changes will be investigated. Thus, it will be revealed if the minimum value of Heisenberg uncertainty in the relativistic domain is still $\hbar/2$ or changes to some other values.

A generalized uncertainty relation, which gives the correction factor to Heisenberg uncertainty and is used as reference in String Theory, has been proposed to be $\Delta p \Delta x \geq \hbar/2\left[1+\beta(\Delta p)^2+...\right]$[11-14]. However, it is intriguing for the authors to review the uncertainty based on the idea that the Lorentz transformation is the generalization of Galilean transformation [2]. Hence, the Heisenberg uncertainty relation observed based on the Lorentz Transformation will be more general and may be able to show something hidden when it is subjected to Galilean transformation. This approach may be able to give an evident difference between relativistic and non-relativistic quantum domain.

Equivalence of the relatvistic energy equation from the particle picture to the wave picture

In this section, we will begin by making particle-wave picture equivalence from equation $E^2 = p^2c^2 + m_0^2c^2$. Equations for particle picture and the wave picture are written respectively as

$$\gamma^2 m_0^2 c^4 = \gamma^2 m_0^2 v^2 c^2 + m_0^2 c^2 \quad (1)$$

and

$$\hbar^2\omega^2 = \hbar^2 k^2 c^2 + \hbar^2\omega_0^2, \quad (2)$$

where ω is the frequency related to total energy, k is the wave number of relativistic momentum, and ω_0 is the intrinsic frequency, which is related to the rest energy [1,5-8]. All quantities in Eq.(2), that have the dimension of energy and momentum will be expressed, respectively in frequency and wave number. Some of these variables are *dummy* variables, which does not have a particular physical meaning. This technique is done merely to make de Broglie wave analysis at the latter stage easier, because all analysis of uncertainty will be expressed in de Broglie wave picture.

We begin with the term k^2c^2 on in Eq. (2). This term will be replaced by a *dummy* ω^{*2}. At this point, ω^* has no relationship with total energy or rest energy, but only a quantity whose dimension is energy when multiplied by $\hbar$. Hence, Eq. (2) becomes

$$\hbar^2\omega^2 = \hbar^2\omega^{*2} + \hbar^2\omega_0^2 \quad (3)$$

Alternatively, Eq. (2) can be written in the following form

$$\hbar^2 k^{*2} c^2 = \hbar^2 k^2 c^2 + \hbar^2 k_0^2 c^2 \quad (4)$$

Eq. (3) and (4) combined implies the following relation

$$\omega / k^* = \omega^* / k = \omega_0 / k_0 = c \quad (5)$$

which is analogous to the relation for photons, i.e. $\omega / k = c$. Eq. (5) allows us to compare frequency−wave number relation to that of massless particles as the standard form. For massless particles, all the forms in Eq.(5) are reduced to simply one relation, i.e. $\omega / k = c$.

In Eq. (4), the terms $\hbar^2 k^{*2}$ and $\hbar^2 k_0^2$ appear have the dimension of momentum, though both of them do not relate to the de Broglie wavelength. The term $\hbar^2 k_0^2$ is related corresponds to the Compton wavelength [2,9-10] and $\hbar^2 k^{*2}$ is just a quantity whose value equal to $\hbar^2\omega^2/c^2$.

After we apply the technique above, the next step is to make the Lorentz factor appear explicitly for frequency of the first and the second term of Eq.(3) as follows

$$\omega = \gamma\omega_0 \text{ and } \omega^* = \gamma\omega_v \quad (6)$$

Here, ω_v denotes a frequency that corresponds to the particle momentum. The reason for this definition is to put the wave picture and its particle counterpart in an analogous form such that direct comparison can be drawn between both picture. Hence, Eq.(3) can be written as

$$\gamma^2\hbar^2\omega_0^2 = \gamma^2\hbar^2\omega_v^2 + \hbar^2\omega_0^2 . \quad (7)$$

By using the same procedure as Eq.(6), Eq.(4) becomes

$$\gamma^2\hbar^2 k_0^2 c^2 = \gamma^2\hbar^2 k_v^2 c^2 + \hbar^2 k_0^2 c^2 . \quad (8)$$

Here, k_v relates with denotes a wavelength that corresponds to ω_v. Comparing Eq.(8) and Eq.(7) the following relations are inferred,

$$\omega_0 = k_0 c \quad \text{and} \quad \omega_v / k_v = c \ . \tag{9}$$

From the analyses above, we obtain quantities $k_v, k_0, \omega_v, \omega^*$, and k^*. They, which are *dummy* variables. In the next step, these variables will be used to simplify the wave analysis.

After making the equivalences above, we can express relativistic energy and momentum, respectively, in frequency and wave number. Based on Eq.(7) and (8), the Lorentz factor in the wave picture is expressed as

$$\gamma = 1/\sqrt{1-\frac{\omega_v^2}{\omega_0^2}} \quad \text{and} \quad \gamma = 1/\sqrt{1-\frac{k_v^2}{k_0^2}} \ . \tag{10}$$

Hence, relativistic energy and momentum in de Broglie wave picture can be expressed as

$$E = \hbar\omega = \hbar\omega_0 / \sqrt{1-\frac{\omega_v^2}{\omega_0^2}} \tag{11}$$

and

$$p = \hbar k = \hbar k_v / \sqrt{1-\frac{k_v^2}{k_0^2}} \ . \tag{12}$$

Relativistic Energy and Momentum Uncertainty in The Wave Picture ($\Delta\omega$ and Δk)

The uncertainty of relativistic energy ΔE in de Broglie wave picture is obtained by differentiating Eq. (11) with respect to ω_v, which yields

$$\Delta E = \hbar\Delta\omega = \hbar\gamma^3 \frac{\omega_v}{\omega_0}\Delta\omega_v \ . \tag{13}$$

It is equivalent to its particle picture, i.e. $\Delta E = \gamma^3 m_0 v \Delta v$. The uncertainty of relativistic momentum Δp in de Broglie wave picture is obtained by differentiating Eq. (12) with respect to k_v, which yields

$$\Delta p = \hbar\Delta k = \hbar\gamma^3 \Delta k_v \ . \tag{14}$$

It is equivalent to its particle picture, i.e $\Delta p = \gamma^3 m_0 \Delta v$.

Eq. (14) and (16) can be expressed, respectively, in the form $\Delta E = \gamma^3 \Delta K_v$ and $\Delta p = \gamma^3 \Delta p_v$, where ΔK_v and Δp_v are uncertainties that have the same form as the uncertainty of non-relativistic kinetic energy and momentum, respectively. Both ΔK_v and Δp_v can be derived from the formula of non-relativistic kinetic energy and momentum, i.e. $K_v = \frac{\hbar^2 k_v^2}{2m_0} = \frac{1}{2}\hbar\frac{\omega_v^2}{\omega_0}$ and $p_v = m_0 v = \hbar k_v$. The value of ΔK_v and Δp_v must be in the relativistic domain as the consequence of the value of ΔE and Δp on the left hand side. So, we can suppose in the theoretical sense that ΔK_v and Δp_v as if the uncertainty form of non-relativistic energy and momentum for the value $v \approx c$. But in reality, ΔK_v and Δp_v must be in domain that $v \ll c$, correspond to Schrodinger equation.

A comparison of momentum and energy uncertainty with and without the Lorentz factor γ^3 tells that for the same Δv or Δk_v, the change in relativistic momentum Δp will be larger than that of non-relativistic momentum Δp_v. This effect also happens to the total energy ΔE against ΔK_v. Relativistic energy and momentum uncertainty will increase by a factor of γ^3 as the consequence of the increase in rest mass in particle picture.

In order to show the relation between ΔE and Δp, Eq.(13) can be stated into $\Delta E = \hbar\gamma^3 \Delta k_v \left(\frac{k_v}{k_0} c\right)$, and by virtue of Eq. (14), we get the following relation

$$\Delta E = \Delta p\left(\frac{k_v}{k_0}c\right) . \qquad (15)$$

By cancelling $\hbar$, it becomes the derivation of dispersion relation, i.e.

$$\gamma^3 \frac{\omega_v}{\omega_0}\Delta\omega_v = \left(\frac{d\omega}{dk}\right)\gamma^3\Delta k_v \ . \qquad (16)$$

The quantity inside the bracket which relates ΔE in Eq.(13) and Δp in Eq. (14) is the speed of de Broglie wave packet, i.e. $v_g = d\omega / dk$ [1,4,5]. The speed can also be expressed in other forms, i.e. $v_g = \frac{k}{k^*}c = \frac{\gamma k_v}{\gamma k_0}c = \frac{\gamma\omega_v}{\gamma k_0} = \frac{\omega^*}{k^*}$. For simply, if $v \approx c$, so $k_v \approx k_0$ and if $v \ll c$ so $k_v \ll k_0$. Equation .(15) can be formed to be the relation between uncertainty of non-relativistic kinetic energy and uncertainty of non-relativistic momentum by cancelling the Lorentz factor, i.e.

$$\Delta K_v = \Delta p_v\left(\frac{k_v}{k_0}c\right), \qquad (17)$$

then by cancelling $\hbar$, it becomes the derivation of dispersion relation, i.e.

$$\frac{\omega_v}{\omega_0}\Delta\omega_v = \left(\frac{d\omega}{dk}\right)\Delta k_v \qquad (18)$$

The existence of γ^3 as a multiplication factor of $\Delta\omega_v$ and Δk_v in Eq.(16) exhibits the range of ΔE and Δp increase in the wave picture. Consequently the wave packet is built according to Eq. (16) is different than that of Eq. (18). The term inside the bracket on Eq.(16) and (18) is the speed of wave packet (particle), nevertheless the speed value from both equations above, i.e., $\frac{k_v}{k_0}c$ are still close to c. The difference between Eq.(16) and (18) is just within the range of frequency and wave number's uncertainty as large as $\gamma^3\Delta\omega_v$ and $\gamma^3\Delta k_v$. So, the form of wave packet without the Lorentz factor i.e Eq. (18) is more appropriate to describe non-relativistic condition, although here the value of v is still close to c. The form of wave packet without the Lorentz factor will has been truly consistent describes non-relativistic particle if the value of v has been made to be smaller-smaller than the speed of light as it should be the purpose of ΔK_v dan Δp_v themselves.

So far, comparison between relativistic and non-relativistic wave packet is understood without considers comparison of the space uncertainty Δx, i.e The comparison is still made based on the same Δx. On the next section, we will show that the Lorentz factor can affect the space uncertainty Δx.

Position and Time Uncertainty under Lorentz Transformation

There is a point which we must explain to the readers about the uncertainty terminology of x and t. In quantum field theory, x and t are viewed as a parameter, hence both of them are not properties of particle. The Δt and Δx respectivelly show how long and how far the state changes correspond to the arbitrary observable Q [1,3]. These process are described as follow:

$$\Delta t = \frac{1}{\langle\frac{dQ}{dt}\rangle}\Delta Q \quad \text{and} \quad \Delta x = \frac{v_g}{\langle\frac{dQ}{dt}\rangle}\Delta Q \qquad (19)$$

Then Δx means the space as large as we can observe the wave packet along this and Δt as the duration when the wave packet as large as Δx is moving. But in the context of Heisenberg uncertainty, we can view both Δx and Δt as if an uncertainty such that can be connected with Δp and ΔE in the sense of uncertainty principle.

If we notice, the relativistic position is $x = \gamma^{-1}x'$ [2].We write that form because position is viewed as a parameter. Nevertheless in the description of de Broglie wave packet, we can view average values for both v and x as $\langle v\rangle$ and $\langle x\rangle$ [1,3]. The average value of speed and position can be related as we have seen according to Ehrenfest theorem [9]. Hence, in this context we can write as $\langle x\rangle = \gamma^{-1}\langle x'\rangle$ (transformation of the central position of wave packet), so properly it will connect with $\langle p\rangle =$

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$\gamma m_0\langle v\rangle$, with γ contains an average value of particle's speed i.e. the group speed of the wave packet [1,3], that is $\gamma =1/\sqrt{1-\frac{\langle v\rangle^2}{c^2}}$. Equivalently, γ=$1/\sqrt{1-\frac{\langle k_v\rangle^2}{k_0^2}}$, hence $\langle p\rangle = \hbar\gamma\langle k_v\rangle$. This average value also prevails for energy-time pair.

The uncertainty of position so that it is consistent with Lorentz transformation must be

$$\Delta x = \gamma^{-1}\Delta x' \tag{20}$$

Here, $\Delta x'$ is interpreted as the proper width of wave packet in S' reference frame which transforms as Δx when it is observed from S reference frame. Consequently, wave packet's width in relativistic case will contract according to Lorentz-Fitzgerald contraction because of factor γ^{-1}, while for non-relativistic wave packet, it will not happen. Then, time uncertainty is written as:

$$\Delta t = \gamma^{-1}\Delta t'. \tag{21}$$

So, based on Eq.(20) and (21) we can understand that $\Delta x = v_g\Delta t$. Here, Δt shows the time which is needed by the contracted wave packet Δx when it is moving through a point in S reference frame according to observer who stays in this frame. This reason comes from the relation $x' = \gamma(x - v_g t)$. If we derive x' only with respect to t, we get the relation $\Delta x' = |-\gamma\Delta t| v_g = \Delta t'\, v_g$. Then from $\Delta x' = \gamma\Delta x$, we get the relation $\Delta x = \Delta t\, v_g$ as the contracted width of wave packet during the time Δt.

If the uncertainty of time is viewed from understanding that the time in S' frame runs slower according to the observer in S frame, so we use inverse Lorentz transformation of time, i.e.,

$$\Delta t = \gamma\Delta t'. \tag{22}$$

Uncertainty of time for relativistic wave packet will dilates as large as γ, the $\Delta t'$ is interpreted as if the proper time. We state like that because the definiton of proper time is $\Delta t' = \Delta x'/c$, not $\Delta t' = \Delta x'/v_g$.

Heisenberg uncertainty under Lorentz transformation

According to the Heisenberg uncertainty relation's format, Eq. (14) and (20) are combined to be $\Delta p\Delta x = \gamma^3 m_0\Delta v\gamma^{-1}\Delta x'$. In the wave picture, momentum-position uncertainty is

$$\hbar\Delta k\Delta x = \gamma^3\hbar\Delta k_v\gamma^{-1}\Delta x'$$

$$\Delta p\Delta x = \gamma^3\Delta p_v\gamma^{-1}\Delta x'.$$

$$\Delta p\Delta x = \gamma^2\hbar/2, \tag{23}$$

Based on $\Delta p = \Delta E/\langle v\rangle$ and $\Delta x = \langle v\rangle\Delta t$, we get $\Delta E\Delta t = \gamma^3 m_0 v\Delta v\gamma^{-1}\Delta t'$. Then, energy-time uncertainty in the wave picture is

$$\hbar\Delta\omega\Delta t = \gamma^3\hbar\frac{\omega_v}{\omega_0}\Delta\omega_v\gamma^{-1}\Delta t'$$

$$\Delta E\Delta t = \gamma^3\Delta K_v\gamma^{-1}\Delta t'.$$

$$\Delta E\Delta t = \gamma^2\hbar/2, \tag{24}$$

But on the other hand, formulation of energy-time uncertainty can also be obtained by taking Δt from Eq.(22), with the consequence that it's energy couple must be taken in the form of Lagrangian according to the rule $\int L dt = -\sqrt{1-\frac{v^2}{c^2}}m_0c^2\tau/\sqrt{1-\frac{v^2}{c^2}}$[15]. In the wave picture, the Lagrangian is $L = -\hbar\omega_0\sqrt{1-\frac{\omega_v^2}{\omega_0^2}}$.

Then we must take the differential form of L so that it is consistent with the rule of Heisenberg uncertainty relation, i.e. $\Delta L = \hbar\gamma\frac{\omega_v}{\omega_0}\Delta\omega_v$. So, the uncertainty relation is

$$\Delta L\Delta t = \gamma\hbar\frac{\omega_v}{\omega_0}\Delta\omega_v\gamma\Delta t',$$

$$\Delta L\Delta t = \gamma\Delta K_v\gamma\Delta t'.$$

$$\Delta L\Delta t = \gamma^2\hbar/2 \tag{25}$$

This form is not equivalent with uncertainty relation of momentum-position (not like $\Delta E\Delta t$). We do not use this later.

Notice that terms on the left hand side of Eq. (23) and (24) show uncertainty relation which are relativisticly consistent. While terms on the right hand side show uncertainty relation which are non-relativistic consistent (without Lorentz factor) i.e. $\Delta p_v \Delta x'$, $\Delta K_v \Delta t'$. So, they are compatible with the Galilean transformation with the minimum limit is $\hbar/2$. It is reasonable because if we take $v \ll c$, value $\gamma \approx 1$, hence Eq.(23) to be $\Delta p \Delta x \approx m_0\, \Delta v \Delta x' = \hbar/2$ (so for Eq.(24) and (25)). It shows that the form of Heisenberg uncertainty for $v \ll c$ approximates the form of Heisenberg uncertainty when the relativistic effect is not yet entered to the quantum domain, as if that the form is derrived directly from equations $\langle p \rangle = m\langle v \rangle$ and $\langle \mathrm{x} \rangle = \langle \mathrm{x}' \rangle$ in non-relativistic limit [1,3].

Based on this analysis, we can see that magnification of the Heisenberg uncertainty is as the natural consequence of the Lorentz transformation, and not because of intervention in the measurement process. The phase space will be larger if the speed of particle close to the speed of light. It means that the minimum limit of Heisenberg uncertainty in describing particle will be different between relativistic and non-relativistic domain. While on reference [4], the Lorentz factor does not appear in uncertainty relation, although it is in the relativistic domain. For massless particle, Eq.(23) and (24) revert to ordinary Heisenberg uncertainty like on reference [16].

In some literatures, there have been confirmed that the Heisenberg uncertainty for relativistic particle is $\Delta p \Delta x = m_0 c \lambda_c = \hbar$ [9,10]. It means that our ability to localize a particle is impossible more accurate than it's Compton wavelength, because the change of particle momentum is impossible larger than $m_0 c$. If we notice Eq.(23) in the particle picture, certainly there is a part $m_0 \Delta v \Delta x'$ whose value equal to $\hbar/2$, with $\sqrt{\langle v^2 \rangle - \langle v \rangle^2} = \sqrt{\langle \Delta v^2 \rangle}$ and $\sqrt{\langle x'^2 \rangle - \langle x' \rangle^2} = \sqrt{\langle \Delta x'^2 \rangle}$ If $\Delta v \ll c$, so $\Delta x'$ reaches the value as large as it can. In this situation, two possibility can occur, i.e. $v_g \approx c$ or $v_g \ll c$. For $v_g \approx c$, the Lorentz factor is significant, so the uncertainty relation is Eq.(23). Then for $v_g \ll c$, the Lorentz factor is not significant, so the uncertainty is $\Delta p \Delta x = m_0 \Delta v \Delta x'$. On the other side, if $\Delta v \approx c$, so $\Delta x' \approx \lambda_c$. In this situation, $v_g = 0$. Hence, Eq.(23) becomes $\Delta p \Delta x = m_0 c \lambda_c = \hbar/2$, or we can write $\hbar/2$ just as $\hbar$. Further, because of $v_g = 0$ in the energy-time uncertainty, i.e. $\Delta E \Delta t = m_0 v_g \Delta v \Delta t = \hbar/2$, it gives consequence that $\Delta E = 0$ and $\Delta t = \infty$. It seems contradictive with $\Delta E \Delta t = m_0 c^2 \tau = \hbar$ [9,10]. Nevertheless, we can see that energy relation $E = K + m_0 c^2$, imply that $E = m_0 c^2$ when $v_g = 0$. Hence we can replace $\Delta E = m_0 c^2$, and obviously $\Delta t = \tau$. It means that although uncertainty of energy which is related with velocity is zero, but the rest energy of particle does still exist. The τ can be interpreted as intrinsic periode that remind us to intrinsic frequency ω_0 in Eq.(2). We give the terminology for the left hand side of Eq.(23) and Eq.(24) as the coordinate Heisenberg uncertainty while the hand side whose value are always $\hbar/2$ are the proper Heisenberg uncertainty. It is like the concept of proper length/time and coordinate length/time.

In the quantum field theory, we do not find commutational relation in the context of (p-x) and (E-t) but we use the relation $[\Phi(x,t), \pi(x',t)] = i\hbar\delta(x-x')$ (for example:Klein-Gordon equation) [17,18]. Nevertheless, it does not mean that uncertainty relation in the context of (p-x) and (E-t) cannot be used again. Both relations can still be used by the following convention:

Table 1 Commutational relation and non-relativistic uncertainty.

Relation	Momentum – Position	Energy-Time
Commutation	$[\mathbf{p},\mathbf{x}] = i\hbar$	-
Heisenberg Uncertainty	$\Delta p \Delta x \geq \hbar/2$	$\Delta E \Delta t \geq \hbar/2$

Table 2 Commutational relation and relativistic uncertainty.

Relation	Momentum-Position	Energy-Time
Commutation	-	-
Heisenberg Uncertainty	$\Delta p \Delta x \geq \gamma^2 \hbar/2$	$\Delta E \Delta t \geq \gamma^2 \hbar/2$

In non-relativistic quantum theory, x and p are operators. So, both of them can be formed to be $[\mathbf{p},\mathbf{x}] = i\hbar$, but for E dan t cannot be formed like that because t is a parameter. Although there is no commutational relation for E and t, both of them can still be formed in the

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uncertainty relation like p and x [2,3]. In the relativistic domain, quantity x and t are a parameter. Both of them are not properties of the particle [17,18], so neither (E-t) nor (p-x) has the commutational relation. Nevertheless, uncertainty relation for both couples does exist. They must be written as $\Delta p\Delta x \geq \gamma^2\hbar/2$ and $\Delta E\Delta t \geq \gamma^2\hbar/2$. It is as the consequence of keeping the consistence of Lorentz transformation. It means that the Klein-Gordon and Dirac solution should follow this condition.

CONCLUSION

De Broglie wave analysis of Heisenberg uncertainty relation under Lorentz transformation yields $\Delta p\Delta x \geq \gamma^2\hbar/2$ and $\Delta E\Delta t \geq \gamma^2\hbar/2$ for the mass particle. The minimum limit of Heisenberg uncertainty principle is $\gamma^2\hbar/2$ if the group velocity of the wave packet closes to the speed of light. It confirms that Heisenberg uncertainty principle does not invarian based on the Lorentz transformation.

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